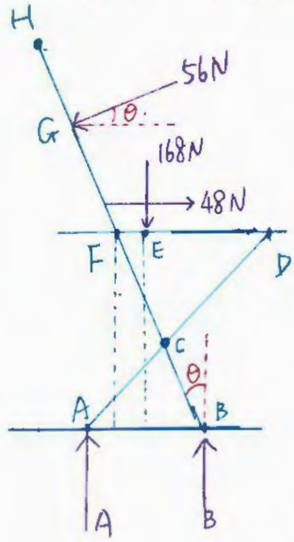
The Solutions of 1st Midterm.

Problem 1.

(a)



entire free-body diagram.

free body diagram 2%

eq. of equilibrium.

$\Sigma F_x = 0$ (→ 為正)

$\Sigma F_y = 0$ (↑ 為正)

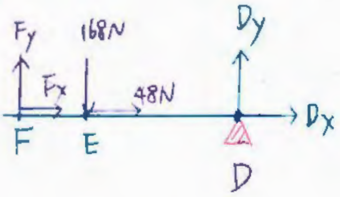
$\Sigma M_B = 0$ (↺ 為正)

$$\Rightarrow \begin{cases} 48 - 56\cos\theta = 0 \\ A + B - 168 - 56\sin\theta = 0 \\ 56 \times \left(0.3 + \dfrac{0.5}{\cos\theta}\right) + 168 \times 0.2 - 48 \times 0.5 - A \times 0.4 = 0. \end{cases}$$

eq. of equilibrium 2%

$$\Rightarrow \begin{cases} \cos\theta = \dfrac{48}{56} \Rightarrow \theta \doteqdot 31.00^\circ \\ A = 147.67\ N \\ B = 49.17\ N. \end{cases}$$

free-body diagram of FD



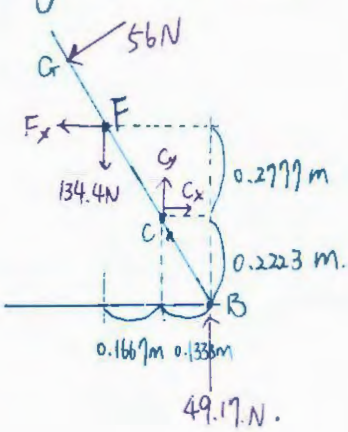
free body diagram 3%

eq. of equilibrium :

eq. of equilibrium 2%

$$\begin{cases} F_x + 48 + D_x = 0 \\ F_y - 168 + D_y = 0 \\ 168 \times 0.4 - F_y \times 0.5 = 0 \ (\Sigma M_D = 0) \end{cases} \Rightarrow \begin{cases} D_x = -48 - F_x \\ F_y = 134.4\ N. \\ D_y = 33.6\ N. \end{cases}$$

free-body diagram of HB.



free body diagram 3%

eq. of equilibrium. :

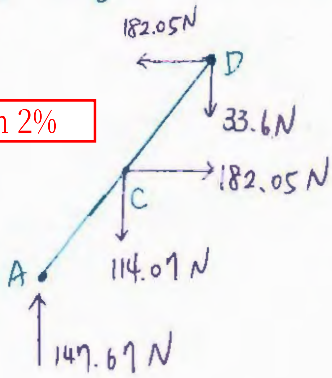
$$\begin{cases} -56\cos\theta - F_x + C_x = 0 \\ 49.17 + C_y - 134.4 - 56\sin\theta = 0 \\ 56 \times \left(0.3 + \dfrac{0.1667}{\sin\theta}\right) + F_x \times 0.2777 + 134.4 \times 0.1667 + 49.17 \times 0.1333 = 0 \\ (\Sigma M_C = 0) \end{cases}$$

$$\Rightarrow \begin{cases} F_x = -230.05\ N \\ C_y = 114.07\ N \\ C_x = -182.05\ N \end{cases}$$

→ 代入 $D_x = -48 - F_x = 182.05\ N$

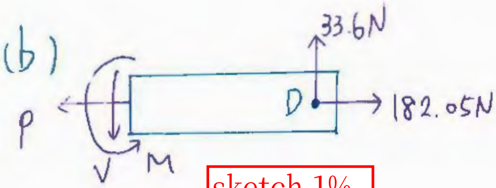
eq. of equilibrium 2%

free-body diagram of AD.



free body diagram 2%

(b)



sketch 1%

by eq. of equilibrium:

$$\begin{cases} P - 182.05 = 0 \\ 33.6 - V = 0 \\ M + 33.6 \times 0.25 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} P = 182.05\ N \\ V = 33.6\ N \\ M = -8.4\ N\cdot m \end{cases}$$ #

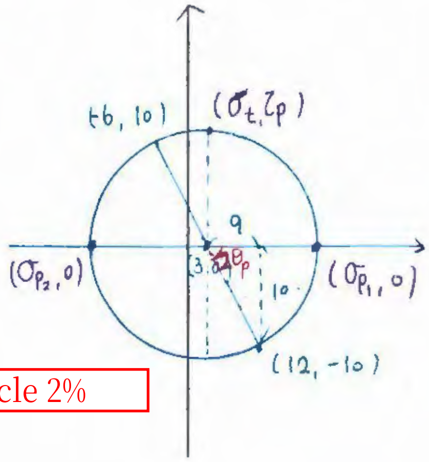
eq. of equilibrium 3%

Problem 2.

(a)

$a = \frac{-6+12}{2} = 3.$

$R = \sqrt{9^2 + 10^2} = \sqrt{181} = 13.45.$



Mohr's circle 2%

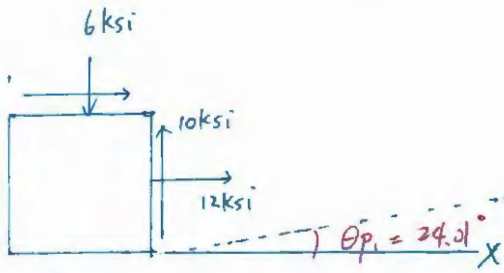
$\tau_{max} = \tau_p = R = 13.45$ ksi 2%

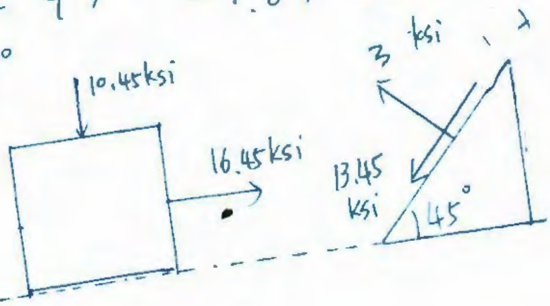
$\sigma_{p_1} = 3 + R = 3 + 13.45 = 16.45$ ksi 2%

$\sigma_{p_2} = 3 - R = 3 - 13.45 = -10.45$ ksi 2%

$\theta_{p_1} = \frac{1}{2}\tan^{-1}\left(\frac{10}{9}\right) = 24.01^\circ$ (ccw) 2%

$\theta_{p_2} = -65.99^\circ$





element 5%

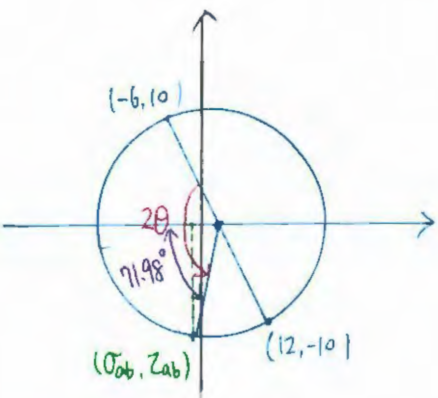
(b)

由正y平面逆時針転60°. ⇒ Mohr's circle上転120°.

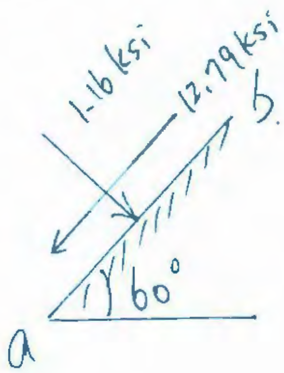
$120^\circ - 48.02^\circ = 71.98^\circ$

$\sigma_{ab} = 3 - 13.45\cos 71.98^\circ = -1.16$ ksi 3%

$\tau_{ab} = 13.45 \times \sin 71.98^\circ = 12.79$ ksi 3%



Mohr's circle 4%



sketch 5%

Problem 3.

$$\delta_{DE} = \varepsilon_{DE} L_{DE} = 0.0006\,(3) = 0.0018\,\text{ft} = 0.0216 \text{ in.}$$

(a)

$$\delta_{AB} = \frac{8}{3}\delta_{DE} = \frac{8}{3}(0.0216) = 0.0576 \text{ in}$$

$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{0.0576}{4\,(12)} = 0.0012 \text{ in/in} = 1200\ \mu\text{in/in}$$ 8%

(b)

$$\delta_{AB} = \frac{8}{3}\delta_{DE} - \text{clearance} = \frac{8}{3}(0.0216) - 0.01 = 0.0476 \text{ in.}$$

$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{0.0476}{4\,(12)} = 0.0009917 \text{ in/in} = 991.7\ \mu\text{in/in.}$$

10%

(c)

$\because$ $\delta_{AB} = \frac{8}{3}\delta_{DE} - \text{clearance} \leq 0.0576$ in.

So if clearance becomes 0.06 in, δ_{AB} would be zero

or no strain would occur. 不會有壓縮！ 7%

Problem 4.

a. $\varepsilon_a = \varepsilon_x = 800 \ \mu in/in.$ $\quad \varepsilon_b = \varepsilon_{120^\circ} = 960 \ \mu in/in$

$\varepsilon_c = \varepsilon_{240^\circ} = 800 \ \mu in/in$ $\quad \nu = 0.33$

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta.$$

$$\varepsilon_b = (800)\cos^2(120^\circ) + \varepsilon_y \sin^2(120^\circ) + \gamma_{xy}\sin(120^\circ)\cos(120^\circ) = 960$$

$$\varepsilon_c = (800)\cos^2(240^\circ) + \varepsilon_y \sin^2(240^\circ) + \gamma_{xy}\sin(240^\circ)\cos(240^\circ) = 800$$

$$0.75\,\varepsilon_y - 0.43301\,\gamma_{xy} = 760.$$

$$0.75\,\varepsilon_y + 0.43301\,\gamma_{xy} = 600$$

$\varepsilon_y = 906.667 \ \mu in/in$

$\gamma_{xy} = -184.752$ rad.

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{1}{2}\tan^{-1}\frac{(-184.752)}{(800)-(906.667)} = 30.00^\circ, \ -60.00^\circ$$

when $\theta_p = 30^\circ$

$$\varepsilon_n = (800)\cos^2\theta_p + (906.667)\sin^2\theta_p + (-184.752)\sin\theta_p\cos\theta_p$$

$$= 746.667 \ \mu in/in = \varepsilon_{p2}$$

$$\varepsilon_{p1} = \varepsilon_x + \varepsilon_y - \varepsilon_{p2} = 960 \ \mu in/in.$$

$$\varepsilon_{p3} = \varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = \frac{-0.33}{1-0.33}[(800)+(906.667)] = -840.597 \ \mu in/in$$

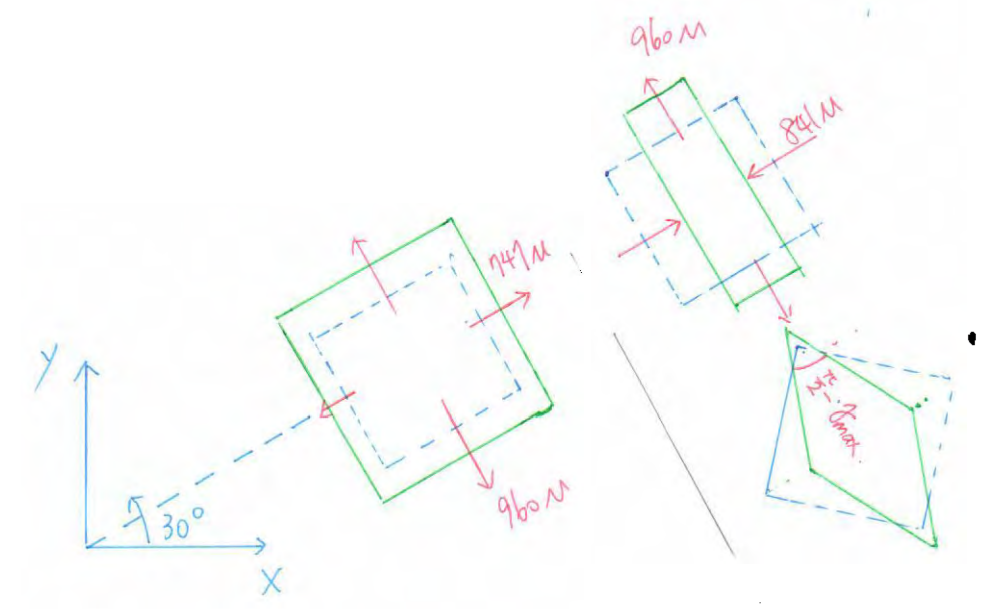
$\varepsilon_{p1} = 960 \ \mu in/in$ 2%

$\varepsilon_{p2} = 747 \ \mu in/in$ 2%

$\varepsilon_{p3} = -841 \ \mu in/in$ 2%

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 213 \ \mu rad.$ 2%

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 1801 \ \mu rad.$ 2%



sketch 5%

(b) $\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$

$= 800 \cos^2(200°) + 906.667 \sin^2(200°) + (-184.752) \sin(200°) \cos(200°)$

$= 706.418 + 106.060 - 59.378$

$= 753.13$ μin/in $\boxed{10\%}$